Unit 5 Trigonometry Review

Instructions: These problems to help you prepare for the test. Use your notes to complete problems.

1. Draw two special triangles.

2. On the unit circle below, mark an arbitrary point (x, y) in quadrant I. Prove why \( x = \cos(\theta) \) and \( y = \sin(\theta) \). Hint, drawing a right triangle will help.

Using the unit circle, complete the table.

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>( \pi \over 2 )</th>
<th>( \pi )</th>
<th>( 3\pi \over 2 )</th>
<th>( 2\pi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \cos(\theta) )</td>
<td>1</td>
<td>o</td>
<td>-1</td>
<td>o</td>
<td>1</td>
</tr>
<tr>
<td>( \sin(\theta) )</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>0</td>
</tr>
</tbody>
</table>
3. Find the exact value of the following.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>a. $\cos(120°)$</td>
<td>b. $\sin(330°)$</td>
</tr>
<tr>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>Q·IV</td>
<td>Q·IV</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>c. $\tan\left(\frac{3\pi}{4}\right)$</th>
<th>d. $\cos\left(\frac{\pi}{2}\right)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tan\left(45°\right)$</td>
<td>$\cos\left(90°\right)$</td>
</tr>
<tr>
<td>$1$</td>
<td>$0$</td>
</tr>
<tr>
<td>Q·IV</td>
<td>Q·IV</td>
</tr>
</tbody>
</table>

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<thead>
<tr>
<th>e. $\sec(315°)$</th>
<th>f. $\csc(270°)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\sqrt{2}}{2}$</td>
<td>$-1$</td>
</tr>
<tr>
<td>Q·IV</td>
<td>Q·IV</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>g. $\cot\left(\frac{5\pi}{6}\right)$</th>
<th>h. $\sin(2\pi)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\sqrt{3}}{3}$</td>
<td>$0$</td>
</tr>
<tr>
<td>Q·III</td>
<td>Q·II</td>
</tr>
</tbody>
</table>

<p>| | |</p>
<table>
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<th></th>
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</table>
4. If $\theta$ is an angle in standard position and $P(-3, -4)$ is a point on the terminal side of $\theta$, what is the value of $\sin \theta$?

\[ \sin \theta = \frac{b}{h} = \frac{-4}{5} \]

5. If the terminal side of angle $\theta$, in standard position, passes through point $(4, -3)$, what is the value of $\sin \theta$?

\[ \sin \theta = \frac{4}{5} \]

6. Point $A\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$ is on the unit circle whose center is the origin. If $\theta$ is an angle in standard position whose terminal ray passes through point $A$, what is the value of $\cos \theta$?

\[ \cos \theta = x = -\frac{\sqrt{2}}{2} \]

7. A circle centered at the origin has a radius of 8 units. The terminal side of an angle, $\theta$, intercepts the circle in Quadrant IV at point C. The x-coordinate of point C is 5. What is the value of $\tan \theta$?

\[ \tan \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{y}{x} = \frac{-\sqrt{15}}{5} \]

8. Determine the exact value of $\sec P$ is $P$ is an angle in standard position and its terminal side passes through the point $(5, -4)$. Write an answer in radical form.

\[ \sec P = \frac{h}{a} = \frac{\sqrt{41}}{5} \]
9. Sketch each of the following equations on the graph below.
   a. $y = 2 \cos(x) - 1$
   b. $y = -\sin(x) + 3$

10. State the amplitude, midline, frequency and period of the following.
    a. $y = -1.5 \sin(2x) + 3$
        amplitude: 1.5
        midline: 3
        frequency: 2
        period: $\frac{2\pi}{2} = \pi$
    b. $y = 3.2 \cos\left(\frac{2\pi}{3}x\right) - 1$
        amplitude: 3.2
        midline: -1
        frequency: $\frac{2\pi}{\frac{2\pi}{3}} = \frac{3}{2}$
        period: $\frac{2\pi}{\frac{3}{2}} = \frac{4\pi}{3}$

11. State the range of each of the following sinusoidal functions in interval form.
    a. $y = -5 \sin(x) - 4$
        $-5 - 5 = -10$
        $-5 + 5 = 0$
        $[-10, 0]$
    b. $y = 3 \cos\left(\frac{2\pi}{3}(x - 1.3)\right) - 2$
        $-2 - 3 = -5$
        $-2 + 3 = 1$
        $[-5, 1]$
12. The Ferris wheel at the landmark Navy Pier in Chicago takes 7 minutes to make one full rotation. The height, \( h \), in feet, above the ground of one of the six-person cars can be modeled by \( h(t) = 70 \sin \left( \frac{2\pi}{7} (t - 1.75) \right) + 80 \), where \( t \) is time, in minutes. Using \( h(t) \) for one full rotation, find the car’s minimum height.

\[
80 - 70 = \sqrt{10} \hfill
\]

13. The heights of the ties can be described using a sinusoidal model of the form \( y = A \cos (Bx) + C \). If high tides are separated by 12 hours, determine the period and frequency.

- \( \beta = 12 \)
- \( \beta = \frac{2\pi}{12} \)
- \( \beta = \frac{\pi}{6} \)

14. The voltage used by most households can be modeled by a sine function. The maximum voltage is 120 volts, and there are 60 cycles every second. Write an equation that represents the value of the voltage at it flows through the electric wires.

a. Determine the amplitude.

\[
\frac{\text{Max} + \text{Min}}{2} = \frac{120 + 0}{2} = 60, \quad d = 60 \quad \frac{120 - 60}{60} = 60 \quad \therefore \quad a = 60
\]

b. Determine the period and frequency.

\[
\frac{60 \text{ cycles}}{1 \text{ sec}} = \frac{1}{x} \quad \text{such that} \quad \beta = \frac{1}{60}, \quad b = 120 \pi \quad \therefore \quad 2\pi = \beta \quad 2\pi = \frac{\pi}{60} \quad b = 120 \pi
\]

c. Write an equation.

\[
V(t) = 60 \sin \left( 120 \pi x \right) + 60
\]
15. Find the inverse of the following functions.

a. \( f(x) = 4x - 3 \)
   \[ y = 4x - 3 \]
   \[ x = 4y - 3 \]
   \[ x + 3 = 4y \]
   \[ \frac{x + 3}{4} = y \]
   \[ f^{-1}(x) = \frac{x + 3}{4} \]

b. \( g(x) = 2x^2 - 5 \)
   \[ y = 2x^2 - 5 \]
   \[ x = 2y^2 - 5 \]
   \[ x + 5 = 2y^2 \]
   \[ \sqrt{\frac{x + 5}{2}} = y \]

16. For the following problems, 
\[ f(x) = 2x - 1 \]
\[ g(x) = 2x^2 + 3 \]

Solve:

a. \((f + g)(2)\)
\[ f(2) = 2(2) - 1 = 3 \]
\[ g(2) = 2(2)^2 + 3 = 11 \]
\[ 3 + 11 = 14 \]

b. \(f(g(1))\)
\[ g(1) = 2(1)^2 + 3 = 5 \]
\[ f(5) = 2(5) - 1 = 9 \]

17. For \(f(x)\) and \(g(x)\) determine whether the relation is a function or not. Explain.

\[ f(x) \text{ - FUNCTION, Passes VLT} \]
\[ g(x) \text{ - NOT A FUNCTION, x VALUES REPEAT} \]

<table>
<thead>
<tr>
<th>( x )</th>
<th>( g(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>-1</td>
</tr>
<tr>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>8</td>
<td>11</td>
</tr>
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