Unit 1 Functions and Their Inverse
Review Packet

Do Now Space:

Part 1: The list of questions covers all the materials we have learned in Unit 1. Please use your previous notes to answer the following questions.

1. When Do We Classify a Relation as a Function?
   a. When each _____________ has only one ___________ value
   b. Looking at a graph, when it passes _______________________________ (or VLT)
   c. Looking at a map, if there is only one ________ coming from any _________ value

2. What does it mean for a value to be a Root, Minimum, or Maximum of a function?
   a. Root is any point where a function touches the ____________.
   b. Root has the same meaning as ____________ and ____________.
   c. Minimum is the ___________ on a graph and maximum is the ___________ on a graph.

Determine the roots, minimum and/or maximum point of the graph.

Roots: ___________________

Minimum: ___________________

Maximum: ___________________
3. What does it mean for a function to be **Concave Up** or **Down**?
(Draw an example of a graph when its function is concave up or concave down.)

![Concave Up and Concave Down Graphs](image)

4. What does an **Inverse Function** look like on a graph?

   a. It reflects each point contained in the function over the line ___________.
   
   b. In order to find points, you have to flip each point’s __ and __ coordinate.
   
   c. In order to find an inverse function, you have to
      i. First, switch __ and __ variables in your equation.
      ii. Second, ____________.

Example: Find the inverse function for the following functions:

\[
f(x) = 2x^2 - 8
\]

\[
y = 2x^2 - 8
\]

\[
x = 2y^2 - 8
\]

\[
x + 8 = 2y^2
\]

\[
\frac{x}{2} + 4 = \frac{y^2}{2}
\]

\[
y = \sqrt{\frac{x}{2} + 4}
\]

\[
g(x) = \{(−2,3), (0,5), (2,−7), (11,0)\}
\]

\[
g^{-1}(x) = \{(3,−2), (5,0), (−7,2), (0,11)\}
\]

5. How do we use **Operations** like Addition and Subtraction?

   a. There are two methods:
      i. Method 1: \(f(x) + g(x)\)
         In words, evaluate each function for \(x\), then add the results.
      
   ii. Method 2: \((f + g)(x)\)
       In words, add the functions, then evaluate the resulting function for \(x\).
Example: Given \( f(x) \) and \( g(x) \), evaluate the expression when \( x = 2 \).

\[
\begin{align*}
\text{Method 1: } f(2) + g(2) & \quad \text{Method 2: } (f + g)(2) \\
\frac{f(2)}{g(2)} &= \frac{3(2) - 4}{2^2 - 2 + 6} = 2 \\
\frac{g(2)}{g(2)} &= \frac{2^2 - 2 + 6}{2} = 8 \\
f(2) + g(2) &= 2 + 8 = 10 \\
(f + g)(2) &= \frac{3x - 4 + x^2 - x + 6}{x^2 + 2x + 2} = \frac{x^2 - 2x + 2}{10}.
\end{align*}
\]

6. How do we find **Composition of Functions**?

   a. Notation: \( f(g(x)) = (f \circ g)(x) \)

Example: Evaluate \( (f \circ g)(2) \), when \( f(x) = x^2 - 5 \) and \( g(x) = \sqrt{2x} + 7 \).

\[
(f \circ g)(2) = f(g(2)) = f(\sqrt{2 \cdot 2} + 7) = 9 \\
f(9) = 9^2 - 5 = 81 - 5 = 76
\]

7. How do we know what **Transformations** are taking place without graphing the equation?

   Compare \( y = x^2 \) and \( y = a(x - d)^2 + c \)

   a. If \( a > 1 \), the graph is **narrower**.
   
b. If \( 0 < a < 1 \), the graph is **wider**.
   
c. If \( a \) is negative, the graph is reflected along the \( x \)-axis.
   
d. As \( d \) increases, the graph **moves to the right**.
   
e. As \( c \) increases, the graph **moves up**.

Example: Describe the transformations from \( y = |x| \) and \( y = -2|x - 5| + 11 \)

\[
a = -2 \quad d = 5 \quad c = 11
\]

The graph is narrower and reflected along the \( x \)-axis. The graph moves to the right 5 units and up 11 units.
Part II: The following problems are Regents Exam Practice Problems. The Unit test will be similar to this format, so please treat it as if you are taking a test.

1. The image of function $f(x)$ is found by mapping each point on the function $(x,y)$ to the point $(y,x)$. This image is a reflection of $f(x)$ in
   1) the $x$-axis
   2) the $y$-axis
   3) the line whose equation is $y = x$
   4) the line whose equation is $y = -x$

2. The inverse function of $\{(2, 6), (-3, 4), (7, -5)\}$ is
   1) $\{(-2, 6), (3, 4), (-7, -5)\}$
   2) $\{(2, -6), (-3, 4), (7, 5)\}$
   3) $\{(6, 2), (4, -3), (-5, 7)\}$
   4) $\{(-6, -2), (-4, 3), (5, -7)\}$

3. What is the inverse of the function $y = 3x + 2$?
   1) $3y = x + 2$
   2) $x = 3y + 2$
   3) $y = \frac{1}{3}x - 2$
   4) $x = \frac{1}{3}y + \frac{2}{3}$

4. If $f(x) = x^2 - 6$, find $f^{-1}(x)$.

5. If $m = \{(-1, 1), (1, 1), (-2, 4), (2, 4), (-3, 9), (3, 9)\}$, which statement is true?
   1) $m$ and its inverse are both functions.
   2) $m$ is a function and its inverse is not a function.
   3) $m$ is not a function and its inverse is a function.
   4) Neither $m$ nor its inverse is a function.
6. If \( g(x) = \sqrt{x} \) and \( h(x) = x^3 - 1 \), what is \( g(h(4)) \)?

1) 5
2) 7
3) \( \sqrt{11} \)
4) \( \sqrt{63} \)

\[ h(4) = 4^3 - 1 = 63 \]

\[ g(63) = \sqrt{63} \]

7. If \( f(x) = \frac{1}{2} x - 3 \) and \( g(x) = 2x + 5 \), what is the value of \( (g \circ f)(4) \)?

1) -13
2) 3.5
3) 3
4) 6

\[ g(f(4)) = g\left(\frac{1}{2}(4) - 3\right) = g(1) = 2(1) + 5 = 7 \]

8. How does the graph of \( f(x) = 3(x - 2)^2 + 1 \) compare to the graph of \( g(x) = x^2 \)?

1) The graph of \( f(x) \) is wider than the graph of \( g(x) \), and its vertex is moved to the left 2 units and up 1 unit.

2) The graph of \( f(x) \) is narrower than the graph of \( g(x) \), and its vertex is moved to the right 2 units and up 1 unit.

3) The graph of \( f(x) \) is narrower than the graph of \( g(x) \), and its vertex is moved to the left 2 units and up 1 unit.

4) The graph of \( f(x) \) is wider than the graph of \( g(x) \), and its vertex is moved to the right 2 units and up 1 unit.
9. Which equation is represented by the graph below?

\[ y = x^2 - 3 \]
\[ y = (x - 3)^2 \]
\[ y = |x| - 3 \]
\[ y = |x - 3| \]

10. The diagram below shows the graph of \( y = |x - 3| \).

Which diagram shows the graph of \( y = -|x - 3| \)?

1) 
2) 
3) 
4)
11. Which is the equation of a parabola that has the same vertex as the parabola represented by \( y = x^2 \), but is wider?
   1) \( y = x^2 + 2 \)
   2) \( y = x^2 - 2 \)
   3) \( y = 2x^2 \)
   4) \( y = \frac{1}{2} x^2 \)

12. Which graph does not represent a function?

1) 
2) 
3) 
4) 

13. Which statement is true about the graphs of \( f \) and \( g \) shown below?

1) \( f \) is a relation and \( g \) is a function.
2) \( f \) is a function and \( g \) is a relation.
3) Both \( f \) and \( g \) are functions.
4) Neither \( f \) nor \( g \) is a function.
14. If \( f(x) = \frac{1}{2}x - 3 \) and \( g(x) = 2x^2 + 5 \),
   a) What is the value of \( f(g(-1)) \)?
   \[
   g(-1) = 2(-1)^2 + 5 = 7
   \]
   \[
   f(7) = \frac{1}{2}(7) - 3 = \frac{7}{2} - 3 = \frac{7}{2} - \frac{6}{2} = \frac{1}{2}
   \]
   b) What is the value of \( (f \circ g)(2) \)?
   \[
   g(2) = 2(2)^2 + 5 = 13
   \]
   \[
   f(13) = \frac{1}{2}(13) - 3 = \frac{13}{2} - 3 = \frac{13}{2} - \frac{6}{2} = \frac{7}{2}
   \]
15. How does the graph of \( f(x) = 3|x - 2| \) compare to the graph of \( g(x) = |x| \)?
   The graph of \( f(x) \) is narrower and moves to the right 2 units.

16. Given the following functions,
   \[
   f(x) = 3x + 8 \\
   g(x) = 3x^3 - 1 \\
   h(x) = \{(−2, 4), (0, −1), (1, 17), (5, 3), (7, 1)\}
   \]
   a) Find \( f(-2) + g(3) \)
   \[
   f(-2) = 4 \\
   g(3) = 3(3)^3 - 1 = 80
   \]
   \[
   f(-2) + g(3) = 4 + 80 = 84
   \]
   b) Find \( g(h(f(-1))) \)
   \[
   f(-1) = 3(-1) + 8 = 5
   \]
   \[
   g(5) = 3
   \]
   \[
   h(3) = 3(3)^3 - 1 = 81 - 1 = 80
   \]
   \[
   g(h(f(-1))) = g(80) = 3
   \]
   c) Find \( g^{-1}(x) \) (meaning, find the inverse of \( g(x) \))
   \[
   y = 3x^3 - 1
   \]
   \[
   x = 3y^3 - 1
   \]
   \[
   \frac{x + 1}{3} = \frac{3y^3}{3}
   \]
   \[
   \frac{y}{3} = \sqrt[3]{\frac{x}{3} + \frac{1}{3}}
   \]